

ON THE MAXIMUM BINARY FRACTION IN GLOBULAR CLUSTER CORES

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ABSTRACT

We study the evolution of binary stars in globular clusters using a novel approach combining a state-of-the-art population synthesis code with a simple treatment of dynamical interactions in the dense cluster core. We find that the combination of stellar evolution and dynamical interactions (binary–single and binary–binary) leads to a rapid depletion of the binary population in the cluster core. The *maximum* binary fraction today in the core of a typical dense cluster like 47 Tuc, assuming an initial binary fraction of 100%, is only about 5%. We show that this is in good agreement with recent *HST* observations of close binaries in the core of 47 Tuc, provided that a realistic distribution of binary periods is used to interpret the results. Our findings also have important consequences for the dynamical modeling of globular clusters, suggesting that “realistic models” should incorporate much larger initial binary fractions than has usually been done in the past.

Subject headings: binaries: close — binaries: general — methods: n-body simulations — globular clusters: general — globular cluster: individual (NGC 104, 47 Tucanae) — stellar dynamics

1. INTRODUCTION

Binary stars play a fundamental role in the dynamical evolution of globular clusters, providing (through inelastic collisions) the source of energy that supports them against gravothermal collapse (Fregeau et al. 2003b; Gao et al. 1991; Goodman & Hut 1989). In the “binary burning” phase, a cluster can remain in quasi-thermal equilibrium with nearly constant core density and velocity dispersion for many relaxation times, in much the same way that a star can maintain itself in thermal equilibrium for many Kelvin-Helmholtz times by burning hydrogen in its core.

At present, there are very few direct measurements of binary fractions in clusters. However, even early observations showed that binary fractions in globular cluster cores are smaller than in the solar neighborhood (e.g., Cote et al. (1996)). Recent Hubble Space Telescope observations have provided further constraints on the binary fractions in many globular clusters (Bellazzini et al. 2002; Rubenstein & Bailyn 1997). The measured binary fractions in dense cluster cores are found to be *very small*. As an example, the upper limit on the core binary fraction of NGC 6397 is only 5-7% (Cool & Bolton 2002).

All dynamical interactions in dense cluster cores tend to *destroy* binaries (with the possible exception of tidal captures, which may form binaries, but turn out to play a negligible role; see § 3). Soft binaries can be disrupted by any tidal interaction with another passing star or binary. Even hard binaries can be destroyed in resonant binary–binary encounters, which typically eject two single stars and leave only one binary remaining (Mikkola 1983), or produce physical stellar collisions and mergers (Bacon et al. 1996; Fregeau et al. 2003a).

In addition, many binary stellar evolution processes can lead to disruptions (e.g., following a supernova explosion of one of the stars) or mergers (e.g., following a com-

mon envelope phase). These evolutionary destruction processes can also be enhanced by dynamics (e.g., more common envelope systems form as a result of exchange interactions; see Rasio, Pfahl, & Rappaport 2000).

It is therefore natural to ask whether the small binary fractions measured in old globular clusters today result from these many destruction processes, and what the *initial binary fraction* must have been to explain the current numbers. We address these questions in this paper, by performing calculations that combine binary star evolution with a treatment of dynamical interactions in dense cluster cores.

2. METHODS AND ASSUMPTIONS

Our initial conditions are described by the following parameters: total number of stars (single or in a binary), N , initial mass function (IMF), binary fraction, f_b , distribution of binary parameters (period, P , eccentricity, e , and mass ratio, q). We typically adopt standard choices used in population synthesis studies, which are based on available observations for stars in the field and in young star clusters (Sills et al. 2003). For the calculations reported here, we use the following initial conditions. We adopt the IMF of Kroupa (2002), which can be written as a broken power law $dN \propto m^{-\alpha} dm$, where $\alpha = 0.3$ for $0.01 \leq m/M_\odot < 0.08$, $\alpha = 1.3$ for $0.08 \leq m/M_\odot < 0.50$, $\alpha = 2.3$ for $m/M_\odot \geq 0.5$. We consider the mass range $0.05 M_\odot$ to $100 M_\odot$. The initial average stellar mass is then $\langle m \rangle = 0.48 M_\odot$. The binary mass ratio, q , is assumed to be distributed uniformly in the range $0 < q < 1$. This is in agreement with observations for $q \gtrsim 0.2$ (Woitas et al. 2001). The binary period, P , is taken from a uniform distribution in $\log_{10} P$ over the range $P = 0.1\text{--}10^7$ d. The binary eccentricity, e , follows a thermal distribution truncated such that there is no contact binary.

We evolve all stars (single and binary) using the population synthesis code *StarTrack* (Belczynski et al. 2002). The evolution of single stars is based on the analytic fits provided by Hurley et al. (2000), but in-

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cludes a more realistic determination of compact object masses (Fryer & Kalogera 2001). All our calculations use metallicity $Z = 0.001$, appropriate for a cluster such as 47 Tuc. We treat the evolution of stellar collision and binary merger products following the prescription of Hurley et al. (2002). To evolve the cluster population, we consider two timesteps. One is associated with the evolutionary changes in the stellar population, Δt_{ev} , and the other with the rate of encounters, Δt_{coll} (see §3). Δt_{ev} is defined so that no more than 2% of all stars change their properties (mass and radius) by more than 5%. The global timestep for the cluster evolution is taken to be $\Delta t = \min[t_{\text{ev}}, t_{\text{coll}}]$.

Our modeling of the cluster dynamics is highly simplified. We assume that the core number density, n_c , and one-dimensional velocity dispersion, σ , remain strictly constant throughout the evolution. These quantities are input parameters used to calculate dynamical interaction rates in the cluster core (see below). While all globular clusters have $\sigma \sim 10 \text{ km s}^{-1}$, the core density can vary by several orders of magnitude. Here we set $n_c = 10^5 \text{ pc}^{-3}$ for most calculations, representative of a fairly dense cluster like 47 Tuc. In general, n_c is the main “knob” that we can turn to increase or decrease the importance of dynamics. Setting $n_c = 0$ corresponds to a traditional population synthesis simulation, where all binaries and single stars evolve in isolation after a single initial burst of star formation. To model a specific cluster, we match its observed core luminosity volume density ρ_L^0 , central velocity dispersion, and half-mass relaxation time.

The escape speed from the cluster core can be estimated from observations as $v_e = 2.5 \sigma_3$ (Webbink 1985), where σ_3 is the three-dimensional core velocity dispersion. Following an interaction or a supernova explosion, any object that has acquired a recoil speed exceeding v_e is removed from the simulation. For computing interactions in the core, the velocities of all objects are assumed to be distributed according to a lowered Maxwellian (King 1965), with $f(v) = v^2/\sigma(m)^2 (\exp(-1.5v^2/\sigma(m)^2) - \exp(-1.5v_e^2/\sigma(m)^2))$ with parameters $\sigma(m) = (\langle m \rangle/m)^{1/2} \sigma_3$ (assuming energy equipartition in the core) and v_e . In addition, we use σ to impose a cut-off for soft binaries entering the core. Any binary with maximum orbital speed $< 0.1 \sigma_3$ is immediately broken into two single stars (Hills 1990).

In the presence of a broad mass spectrum, the cluster core is always dominated by the most massive objects in the cluster, which tend to concentrate there via mass segregation. As stars evolve, the *composition* of the core will therefore change significantly over time. To model mass segregation in our simulations, we assume that the probability for an object of mass m to enter the core after a time t_s follows a Poisson distribution, $p(t_s) = (1/t_{sc}) \exp(-t_s/t_{sc})$, where the characteristic mass-segregation timescale is given by $t_{sc} = 10 (\langle m \rangle/m) t_{\text{rh}}$ (Fregeau et al. 2002). Here t_{rh} is the half-mass relaxation time, which we assume to be constant for a given cluster. We fix $\langle m \rangle = 2 M_\odot$, as this value gives, in our model, the best fit for the ratio of core mass to total cluster mass in 47 Tuc.

All objects are allowed to have dynamical interactions after they have entered the cluster core. We use a simple Monte-Carlo prescription to decide which pair of objects

TABLE 1. REFERENCE MODELS.

Model	$\log n_c$	$\log t_{\text{rh}}$	$f_{\text{b,c}}$	$f_{0.5}$	f_{wd}	f_{b}
1	5.0	9.0	0.08	0.07	0.04	0.65
D3	3.0	9.0	0.21	0.25	0.10	0.70
D4	4.0	9.0	0.14	0.15	0.07	0.66
D6	6.0	9.0	0.03	0.02	0.02	0.66
T8	5.0	8.0	0.10	0.04	0.03	0.22
T10	5.0	10.	0.06	0.06	0.04	0.85
B05	5.0	9.0	0.05	0.05	0.05	0.34
ND	5.0	0.0	0.90	0.77	0.50	0.90

NOTE. — n_c is the core number density in pc^{-3} (assumed fixed), t_{rh} is the half-mass relaxation time in yr, $f_{\text{b,c}}$ is the binary fraction in the core, $f_{0.5}$ is the binary fraction for non-degenerate stars more massive than $0.5 M_\odot$, f_{wd} is the binary fraction among white dwarfs and f_{b} is the overall binary fraction in the cluster. ND is the model with no dynamical interactions (field population), where all stars are assumed to be in the core from the beginning. Values for all binary fractions are given at 14 Gyr.

actually have an interaction during each timestep. We consider separately binary–binary and binary–single interactions, as well as single–single encounters (tidal captures and collisions). Tidal captures are treated using the approach described in Portegies Zwart & Meinen (1993). If the pericenter distance is less than twice the sum of the stellar radii, the encounter is treated as a physical collision and assumed to lead to a merger. Each dynamical interaction involving a binary is calculated using *Fewbody*, a new numerical toolkit for simulating small- N gravitational dynamics that is particularly suited to performing 3-body and 4-body integrations (Fregeau et al. 2003a; Fregeau & Rappaport 2003). A more detailed description of our Monte-Carlo procedure will be given in (Ivanova et al. 2004).

Initial conditions for all our reference models are given in Table 1. All models have central velocity dispersion $\sigma_1 = 10 \text{ km s}^{-1}$, and initial primordial binary fraction $f_{\text{b},0} = 1$, except for Model B05, which has $f_{\text{b},0} = 0.5$. Our assumed period distribution implies that about 60% of primordial binaries are hard. The initial number of stars is $N = 2.5 \times 10^5$, with the cluster core containing about 1% of the stars initially.

3. RESULTS

First consider our results for a typical dense cluster, in Model 1. Starting with 100% binaries initially, the final core binary fraction (at 14 Gyr), $f_{\text{b,c}}$, is only 8%. This is strikingly low, given that the cluster started with *all* binaries. Decreasing the initial binary fraction, $f_{\text{b},0}$, to a more reasonable (but still large) 50% reduces $f_{\text{b,c}}$ further to 5%, as shown in Model B05. The dependence of $f_{\text{b,c}}$ on $f_{\text{b},0}$ is not linear. This is mainly due to mass segregation: decreasing $f_{\text{b},0}$ also increases the ratio of mean binary mass to mean stellar mass in the cluster, thereby resulting in a higher concentration of binaries in the core. The majority (about 75%) of destroyed binaries were disrupted by close dynamical encounters (or, rarely, following a supernova explosion). Note that some binaries that are initially hard eventually become soft after undergoing significant mass loss due to stellar evolution. About 20% of the destroyed binaries experienced mergers, typically after significant hardening through interactions. The rate of binary destruction by mergers is about

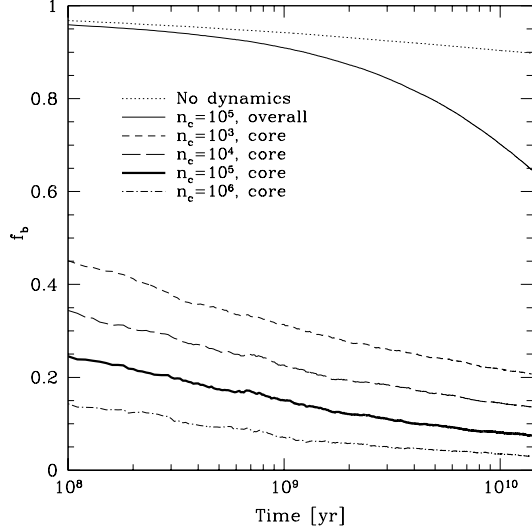


FIG. 1.— Evolution of the core and overall binary fraction in Model 1. Also shown is the evolution with no dynamics (ND) and for different core number densities (D3, D4 and D6).

an order of magnitude higher in this model (Model 1) than for the corresponding field population (Model ND). A few percent of the binaries lost were actually not destroyed but instead were ejected from the cluster as the result of strong encounters. Tidal capture did not play a significant role; the total number of tidal capture binaries formed during the cluster lifetime is less than 1% of the final number of binaries in the core. While the final core binary fraction is extremely low, the overall cluster binary fraction remains high, about 65%, even after 14 Gyr (Fig. 1). Note, however, that the surviving binaries include mainly low-mass systems which never entered the core (about 70% of the initial binaries never entered the core and therefore never had a chance to interact). The average primary mass among binaries remaining outside the core at 14 Gyr is $0.2 M_{\odot}$.

Let us now compare results for different central densities, in Models 1, D3, D4 and D6. The evolution of $f_{b,c}$ for these models is shown in Figure 1. For comparison, the binary fraction for the field case (Model ND) is also presented. As expected, the core binary fraction decreases as n_c increases. However, if one consider the binary fraction for only non-degenerate stars, its behavior is different. In particular, the final binary fraction for non-degenerate stars more massive than $0.5 M_{\odot}$, $f_{0.5}$, is higher than $f_{b,c}$ for the low-density Model D3. Thus $f_{b,c}$ is decreased partially through a lower binary fraction of degenerate objects. Degenerate objects, compared to non-degenerate $0.5 - 0.9 M_{\odot}$ stars, evolved from initially more-massive stars and are more likely to have a more massive companion initially. Their binary destruction rate is much higher, enhanced both by stellar evolution (mass loss and mass transfer at more advanced evolutionary stages, SN explosions in a binary), and by dynamical interactions (large cross-section for encounters).

Next we examine how the half-mass relaxation time affects binary fractions (Models T8 and T10). We see that, surprisingly, the model with shorter relaxation time has a higher core binary fraction. There are two compet-

TABLE 2. MODELS OF SPECIFIC CLUSTERS

Model	$\log \rho_L^o$	$\log n_c$	σ_1	$\log t_{rh}$	$f_{b,c}$	$f_{0.5}$	f_{wd}
NGC3201	2.73 ^a	3.3	5.2	9.2	0.17	0.22	0.12
ω Cen	3.37	4.0	16	10	0.13	0.16	0.06
M3	3.51	3.9	4.8 ^b	8.5	0.16	0.13	0.07
M4	3.82	4.3	4.2	8.82	0.12	0.10	0.07
47Tuc	4.81	5.3	11.5	9.48	0.05	0.04	0.03
NGC6397	5.68	6.5	4.5	8.46	0.03	0.02	0.02

NOTE. — ρ_L^o is the observed core luminosity density in $L_{\odot} \text{pc}^{-3}$, n_c is the core number density in pc^{-3} , σ_1 is the velocity dispersion in km s^{-1} , t_{rh} is the half-mass relaxation time in yr, $f_{b,c}$ is the binary fraction in the core, $f_{0.5}$ is the binary fraction for non-degenerate objects more massive than $0.5 M_{\odot}$, and f_{wd} is the ratio of WD in binaries to the total number of WD. Values for all binary fractions are given at 14 Gyr.

^aunless specified, ρ_L^o and t_{rh} for specific clusters are taken from Harris (1996), σ_1 from Pryor & Meylan (1993).

^bDubath et al. (1997)

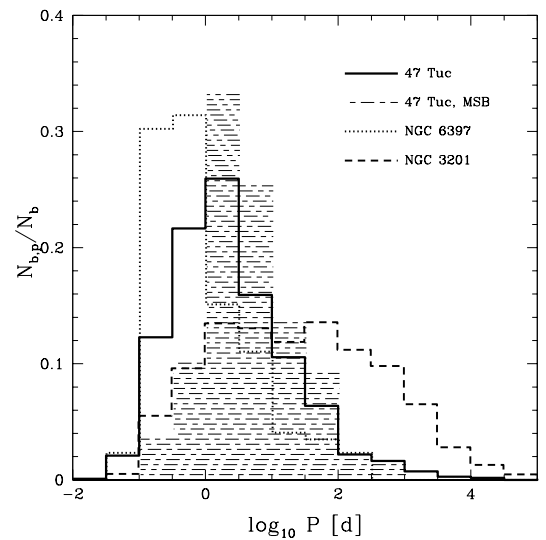


FIG. 2.— The binary period distribution in the models of 47 Tuc, ω Cen, and NGC 6397 at 14 Gyr. The shaded histogram shows the period distribution of binaries containing two MS stars with masses greater than $0.25 M_{\odot}$ in 47 Tuc. N_b is the total number of binaries and $N_{b,p}$ is the number of binaries in the given period range.

ing mechanisms that play a role here: mass segregation, which brings binaries into the core, and dynamical interactions, which destroy binaries in the core. A shorter segregation time increases the rate at which binaries enter the core but also allows less massive binaries to interact. Therefore, the average mass of a binary in the core will be smaller. However, the average time spent by a binary in the core also increases, so more can be destroyed, and the more massive binaries tend to be destroyed first as they have a larger interaction cross section. As a result, in Model T8 $f_{b,c}$ is higher than in T10, although the binary fraction of more massive binaries is smaller.

4. DISCUSSION AND COMPARISON WITH OBSERVATIONS

We performed several simulations with parameters that attempt to match those of specific globular clusters in the Galaxy (Table 2). In all cases the initial binary fraction is 100%, so our results for final core binary

fractions represent upper limits. As before we obtain very low values for $f_{b,c}$. For example, in the 47 Tuc model, $f_{b,c}$ is only 5%. At first glance, this may seem to conflict with observations. In particular, Albrow et al. (2001) derive a binary fraction for the core of 47 Tuc of about 13%, from observations of eclipsing binaries with periods in the range $P \simeq 4\text{--}16\text{ d}$. This estimate was based on an extrapolation assuming a period distribution flat in $\log_{10} P$ from about 2 d to 50 yr. In Figure 2 we show the period distribution of core binaries in our simulation. Note that the period range of eclipsing binaries corresponds to the peak of the distribution, while for larger P it drops rapidly. In particular, for binaries with components more massive than $0.25 M_{\odot}$, the number of systems with periods in the range 2 d to 50 yr is about 7 times smaller than would be predicted by a distribution flat in $\log_{10} P$. Using the observed number of eclipsing binaries and those from our simulation, the adjusted core binary fraction from Albrow et al. (2001) is about 4%, which is consistent with our results. Figure 2 also shows the period distributions for models that represent the clusters NGC 3201 and NGC 6397. For denser clusters, such as NGC 6397, the peak of the distribution shifts toward shorter periods, while for less dense clusters, such as NGC 3201, the distribution peaks at longer periods and is flatter. We performed 3 additional simulations for 47 Tuc, with $f_{b,0} = 0.25, 0.5$ and 0.75 . We found that with decreasing $f_{b,0}$, the adjusted core binary fraction decreases and reaches, e.g., 2.5% for $f_{b,0} = 0.5$.

An alternative estimate of the binary fraction in the core of 47 Tuc is based on observations of BY Dra stars (Albrow et al. 2001). Their estimated core binary fraction, which can be considered a *lower limit*, is approximately 0.8%, 18 times lower than the estimate based on eclipsing binaries. This estimate was based on 31 BY Dra binaries and 5 eclipsing binaries observed in the period range 4–10 d. We analyzed the core binary population in our model in order to identify BY Dra binaries. We adopted the standard definition for a BY Dra binary: primary mass in the range $0.3\text{--}0.7 M_{\odot}$ (see, e.g., Bopp & Fekel (1977)) and period in the range 4–10 d (as for the observed sample in 47 Tuc). The ratio between the total number of binaries and the number of BY Dra systems is found to be 38, 42 and 45 for models with

$f_{b,0} = 1.0, 0.75$ and 0.5 , respectively. With 36 binaries observed in this period range, the total core binary fraction is 2.9%, 3.3% and 3.5% for $f_{b,0} = 1.0, 0.75$ and 0.5 , respectively. Based on these results, we estimate that the initial binary fraction should be at least 0.5, and more likely in the range $f_{b,0} \simeq 0.75 - 1$.

Even more extreme results are obtained for NGC 6397. This cluster is classified observationally as “core collapsed” and therefore may not be well described by our simplified dynamical model. Nevertheless, it is useful to study how the binary fraction in the cluster would have evolved if the very high central density had been constant throughout the evolution. We find that the binary fraction for stars more massive than $0.5 M_{\odot}$ is extremely low: 2% in the core at 14 Gyr with 100% binaries initially. For this cluster, there is a firm upper limit of 3% on the core binary fraction for stars in the mass range $0.45 - 0.8 M_{\odot}$ and for binary mass ratios $q > 0.45$ (Cool & Bolton 2002). For the same ranges in our model we find 2%, in surprisingly good agreement with the observations.

In addition to their implications for the interpretation of observed binary fractions in cluster cores, our results also have important consequences for the theoretical modeling of globular clusters. Indeed, it is clear that “realistic” dynamical simulations of globular cluster evolution should include large populations of primordial binaries, with initial binary fractions in the range $\sim 50\%\text{--}100\%$ (similar to what is usually assumed for the field; see, e.g., Duquennoy & Mayor (1991)). This poses a particular challenge for direct N -body simulations, where the treatment of even relatively small numbers of binaries can add enormous computational costs. For this reason, current direct N -body simulations of star clusters with large initial binary fractions typically have N too small to be considered representative of globular clusters (see, e.g., Portegies Zwart et al. 2003; Wilkinson et al. 2003). However, approximate methods, such as Monte-Carlo, do not suffer from the same limitations, and routinely simulate clusters with reasonably large N ($\sim 3 \times 10^5$) and binary fraction ($\sim 30\%$), yet have not yet included advanced treatments of binary star evolution (see, e.g., Fregeau et al. 2003b, and references therein).

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